U5:L3 Absolute Value Equations

Fill in the following lessons with help from your textbook (pages 380 - 388) or

An **absolute value equation** is an equation that includes the absolute value of an expression involving a Variable.

There are two possible cases:

CASE 1	Expression inside Absolute Value is ⊕ or ∅
CASE 2	Expression inside Absolute Value is ⊖

Solving an Absolute Value Equation

Solve $|x-3| = 7$

Think about the piecewise function:

$$|x-3| = \begin{cases} x-3 & \text{if } x \geq 3 \\ -(x-3) & \text{if } x < 3 \end{cases}$$

$x-3$ if $x \geq 3$ → Case 1 (⊕ OR ∅)

$-(x-3)$ if $x<3$ → Case 2 (⊖)

Case 1

$x-3=7$

$x=10$ ⇒ VERIFY: $|(10)-3|=7$

$|7|=7$ ✓

$7=7$

Case 2

$-(x-3)=7$

$x-3=-7$

$x=-4$ ⇒ VERIFY: $|(-4)-3|=7$

$|-7|=7$

$7=7$ ✓

What this tells you is WHERE the graph of $|x-3|$ is @ 7. This will be @ (10, 7) and (-4, 7)

Graph w/ Desmos to verify!

Remember from Booklet 1 we get this by doing:

$$|2x-5| = 0$$
$$2x-5 = 0$$
$$2x = 5$$
$$x = 5/2$$

Extraneous Solutions

Solve $|2x - 5| = 5 - 3x$

Piecewise Function

$$|2x-5| = \begin{cases} 2x-5, & \text{if } x \geq \frac{5}{2} \\ -(2x-5) & \text{if } x < \frac{5}{2} \end{cases}$$

Case 1

$$2x-5 = 5-3x$$
$$5x = 10$$
$$x = 2$$

BUT $x=2$ does not fit the rule $x \geq 5/2$ so it is EXTRANEOUS

Case 2

$$-(2x-5) = 5-3x$$
$$-2x+5 = 5-3x$$
$$x = 0$$

✓ The value $x=0$ does follow Rule #2 $x < 5/2$

∴ The graph of $y=|2x-5|$ will share ~~the~~ a point with the graph $y=5-3x$ when $x=0$

If you need help solving, or if you want to double check your answers, use your good friend Desmos (the graphing app)!

Absolute Value Equations with No Solution

Solve $|3x - 4| + 12 = 9$

$$|3x-4| + 12 = 9$$
$$|3x-4| = -3$$

☆ An absolute value can never be a ⊖ number!

∴ The graph $y=|3x-4|+12$ will never touch $y=9$

Reminder from Booklet #1:

$0 = |x^2 - 2x|$

$0 = x(x-2)$

$0 = x$ $\qquad$ $0 = x - 2$

$2 = x$

Solving with Quadratic Expressions

Solve $|x^2 - 2x| = 1$

Piecewise Function

$$|x^2 - 2x| = \begin{cases} x^2 - 2x, \text{ if } x \le 0 \text{ or } x \ge 2 \\ -(x^2 - 2x), \text{ if } 0 < x < 2 \end{cases}$$

Case 1

$x^2 - 2x = 1$ $\quad$ ☆ use quadratic formula to solve for x (just like back in U3)!

$x^2 - 2x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{8}}{2} = \frac{\cancel{2} \pm \cancel{2}\sqrt{2}}{\cancel{2}} = 1 \pm \sqrt{2}$$

SIMPLIFIED RADICAL

CHECK: $1+\sqrt{2} = 2.41$ ✓ and $1-\sqrt{2} = -0.41$ ✓ $\quad x \le 0$ or $x \ge 2$ (RULE #1)

Case 2

$-(x^2 - 2x) = 1$

$x^2 - 2x = -1$

$x^2 - 2x + 1 = 0$ $\quad$ ✓ ALL good with Rule #2

$(x-1)^2 = 0$ $\quad$ $0 < x < 2$

$x - 1 = 0$

$x = 1$

Double check with Desmos!

∴ The graph of $y = |x^2 - 2x|$ should have values of $y = 1$ @ $(-0.41, 1)$ and $(1, 1)$

$(2.4, 1)$

Solving with Linear and Quadratic Expressions

Solve $|x-10| = x^2 - 10x$

Piecewise Function

$$|x-10| = \begin{cases} x-10 & \text{if } x \geq 10 \\ -(x-10) & \text{if } x < 10 \end{cases}$$

$0 = |x-10|$

$0 = x-10$

$10 = x$

Case 1

$x - 10 = x^2 - 10x$

$0 = x^2 - 11x + 10$

$0 = (x-10)(x-1)$

$0 = x - 10$ $\qquad$ $0 = x - 1$

$10 = x$ ✓ $\qquad$ $1 = x$ EXTRANEOUS - does not follow Rule #1 $x \geq 10$

Case 2

$-(x-10) = x^2 - 10x$

$-x + 10 = x^2 - 10x$

$0 = x^2 - 9x - 10$

$0 = (x-10)(x+1)$

$x = 10$ is EXTRANEOUS for case 2, but still good for Case #1

$0 = x - 10$ $\qquad$ $0 = x + 1$

$10 = x$ EXTRANEOUS $\qquad$ $-1 = x$ ✓

PRACTICE: Page 389, Questions 4 (a,b,c), 5 (c,d,e), 6 (b,d,e)

U5:L4 Reciprocal Functions

Fill in the following lessons with help from your textbook (pages 392 - 402) or

Reciprocal is a mathematical expression or function so related to another that their product is ___1___; the quantity obtained by dividing the number one by a given quantity.

$$\frac{1}{3} \cdot \frac{3}{1} = \frac{3}{3} = \frac{1}{1} = 1$$

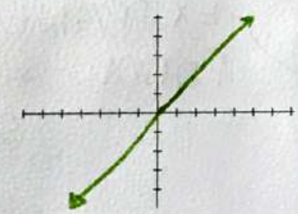
ORIGINAL

RECIPROCAL

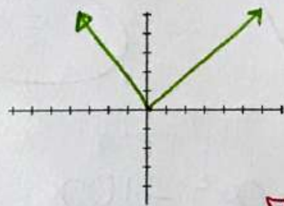
For any non-zero real number (a) the reciprocal of a is ___$\frac{1}{a}$___

For a function $f(x)$, the reciprocal is $\frac{1}{f(x)}$ so long as ___$y \neq 0$___

Regular $y = mx + b$ function



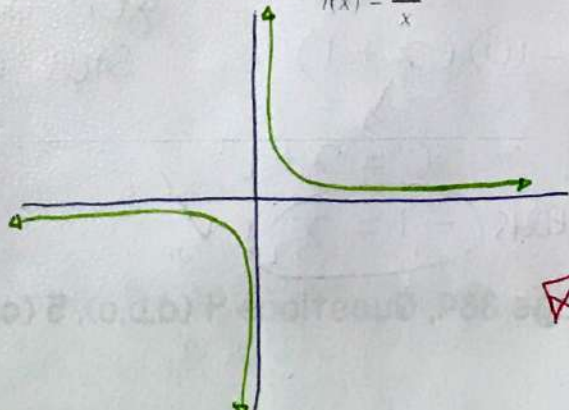
$y = x$
The identity function



$y = |x|$
The absolute value function

Review of Booklet #1 Absolute Value Functions

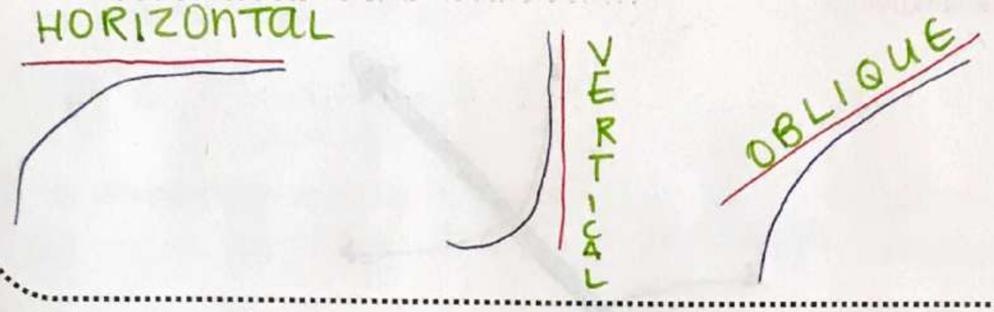
$f(x) = \frac{1}{x}$



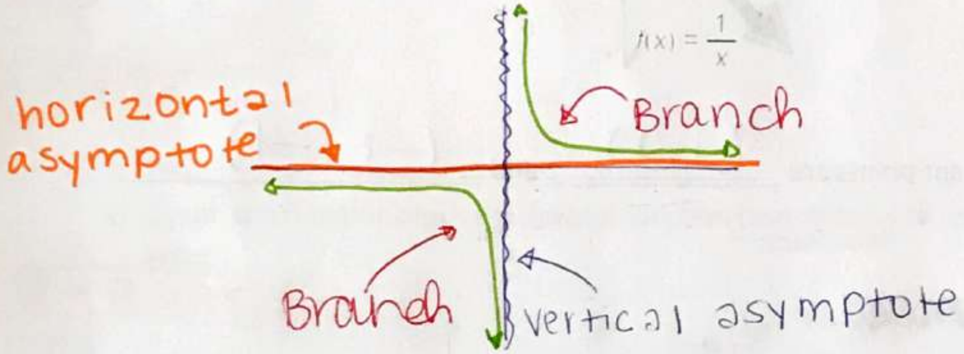
☆ NEW ☆

RECIPROCAL FUNCTION

Reminder: an **asymptote** is a line that continually approaches a given curve but does not meet it at any finite distance.



The graph of $y = \frac{1}{x}$ has two distinct pieces (aka: "**branches**").



These branches are on either side of the vertical and horizontal asymptotes.

This **vertical asymptote** occurs at the **non-permissible values** of the function.

non-permissible means denominator $\neq 0$

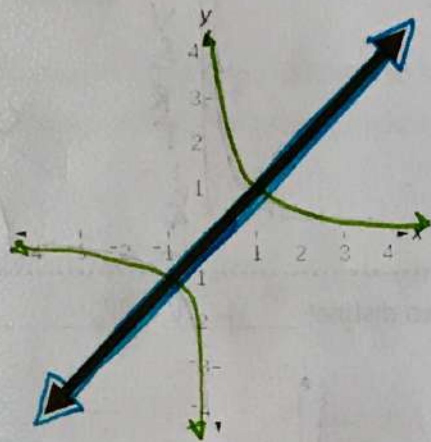
so … $y = \frac{1}{x}$, $x \neq 0$ so vertical asymptote @ $x = 0$

The **horizontal asymptote** exists because $y \neq 0$

The horizontal asymptote describes the behavior of the graph when $|x|$ is very large. When $|x|$ is large, y approaches 0 closely, but never equals 0.

Invariant points are common points between a function and its reciprocal.

If you superimpose the graph of $\boxed{y = x}$ over the reciprocal graph $\boxed{y = \frac{1}{x}}$ you will see two invariant points.



The invariant points are (1, 1) and (−1, −1).

☆ Use Desmos...

Graph $y = -x$ and its reciprocal graph.

Describe how this is different from the above graph:

What are the invariant points?

Reciprocals of Linear Functions

Consider $y = 2x + 5$

a) What is the reciprocal function?

$$y = \frac{1}{2x+5}$$

b) Where will the vertical asymptote be?

@ non-permissible x values (when the denominator $= 0$)

$$2x + 5 = 0$$
$$2x = -5$$
$$x = \frac{-5}{2}$$

c) Graph the function and its reciprocal function (use Desmos to help if you want!)

$\boxed{\text{invariant points}}$ @ ± 1

$2x+5=1$

$2x=-4$ $(-2,1)$

$x=-4/2$

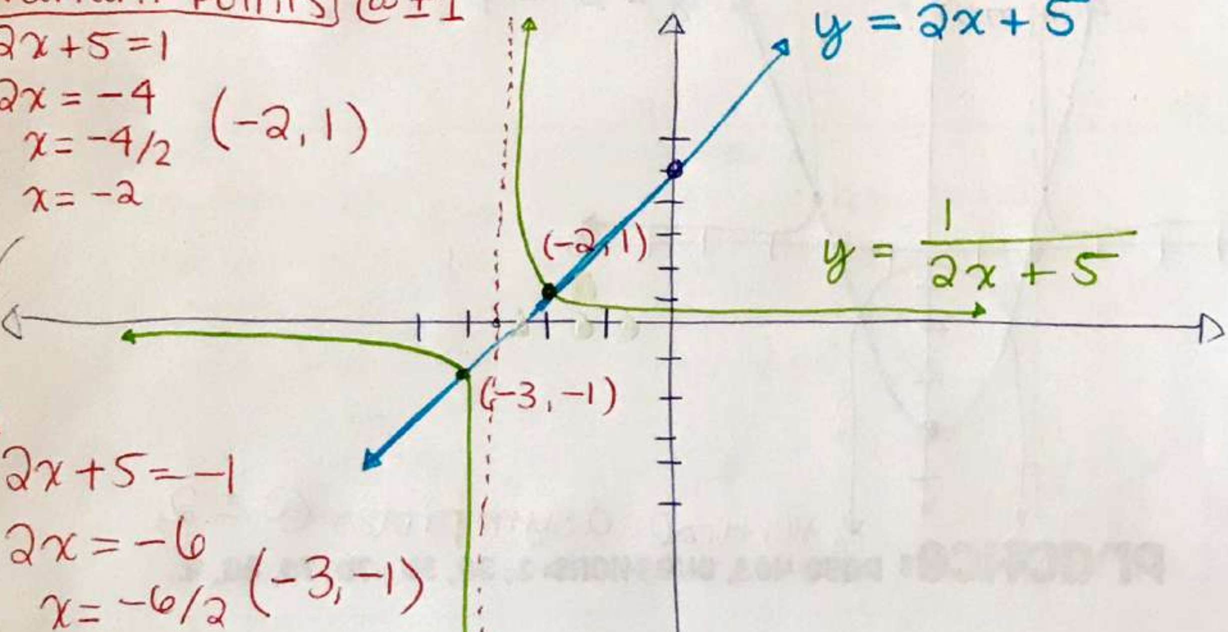
$x=-2$

$2x+5=-1$

$2x=-6$

$x=-6/2$ $(-3,-1)$

$x=-3$



vert. asymptote @ $-5/2$

Graph the reciprocal of a Quadratic Function

Consider $y = x^2 - 4$

a) What is the reciprocal function?

$$y = \frac{1}{x^2 - 4}$$

b) Where will the vertical asymptote be?

@ denominator = 0 (non permissibles)

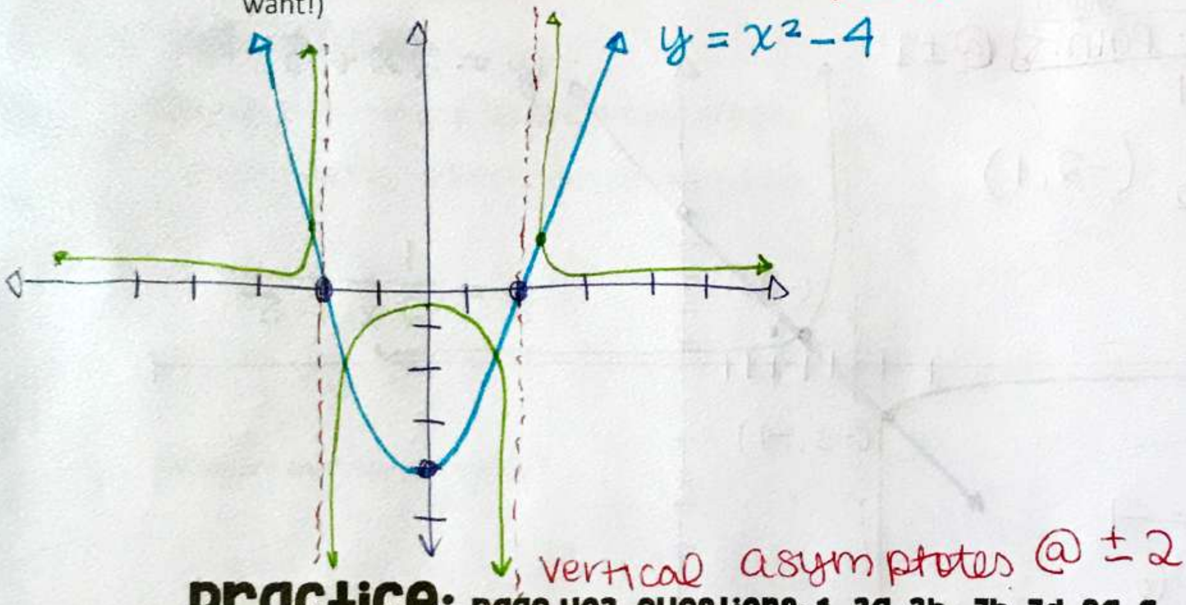
$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$x - 2 = 0$ → $x = 2$

$x + 2 = 0$ → $x = -2$

c) Graph the function and its reciprocal function (use Desmos to help if you want!)



Practice: page 403, Questions: 1, 3a, 3b, 7b, 7d, 8a, 9.