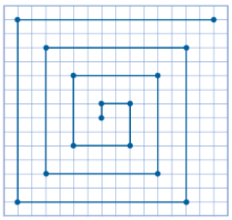
NAME:______________________________

U1:L2 Arithmetic Series



On a separate graph paper, draw a square spiral following these steps:

1) Starting from the middle, draw a line segment 1 unit up.
2) From the end of that segment, draw a 2nd line 1 unit longer than the previous segment towards the right.
3) Continue this pattern 13 times (always following a clockwise rotation).

Record the lengths of this segment as an arithmetic sequence:

- Term
- Length of segment
- Total length of spiral

t_1												
1												
1												

Can you derive a formula to find the total length of the spiral at 20 segments?

$\ell_n = \text{total of } n-1 + t_n$

How about 1000 segments?

What are our strategies for finding this formula? What questions are we asking? What mistakes are we making?

An **arithmetic series** is a sum of terms that form an arithmetic sequence.

EX: for the sequence 2,4,6,8, the arithmetic series is 2+4+6+8

This arithmetic series is represented as S_n ("S sub n") or Σ this symbol is called SIGMA and it means literally *"to sum up"*

$$t_1 + (t_1+d) + (t_1+2d) + (t_1+3d)\ldots$$

$$S_n = \cancel{t_1} + (t_1+d) + \ldots \quad (t_1+(n-2)d) + (t_1+(n-1)d)$$

$$+\ S_n = \cancel{(t_1+(n-1)d)} + (t_1+(n-2)d) \ldots (t_1+d) + \quad (t_1)$$

$$\frac{2S_n}{2} = \frac{(2t_1+(n-1)d) + (2t_1+(n-2)d)\ldots}{2}$$

$$S_n = \frac{n}{2}(2t_1+(n-1)d)$$

We end up with the formula:

$$\boxed{S_n = \frac{n}{2}(2t_1+(n-1)d)}$$

...which can also be written as...

$$\boxed{\sum^{n-1} = \frac{n}{2}(2t_1+(n-1)d)}$$

Because $t_n = t_1 + (n-1)d$ we can substitute this value to also use...

$$S_n = \frac{n}{2}(t_1+t_n)$$

EXAMPLE:

Add up the first ten terms of the sequence: { 1, 4, 7, 10, 13, ... }

+3 +3

$n = 10$

$t_1 = 1$

$d = +3$

$$S_n = \frac{n}{2}(2t_1 + (n-1)d)$$

$$S_n = \frac{10}{2}(2(1) + (10-1)3)$$

$$S_n = 5(2 + 9(3))$$

$$S_n = 5(29)$$

$$\boxed{S_{10} = 145}$$

What is the sum of the first sixteen terms of the arithmetic sequence 1, 5, 9, 13, ... ?

$n = 16$

$t_1 = 1$

$d = +4$

$$S_n = \frac{n}{2}(2t_1 + (n-1)d)$$

$$S_{16} = \frac{16}{2}(2(1) + (16-1)4)$$

$$\sum^{16-1} = 8(2 + (15)4)$$

$$\boxed{S_{16} = 496}$$

What is the sum of the first thirty terms of the arithmetic sequence 50, 45, 40, 35, ... ?

$n = 30$

$t_1 = 50$

$d = -5$

$$S_n = \frac{n}{2}(2t_1 + (n-1)d)$$

$$S_{30} = \frac{30}{2}(2(50) + (30-1)-5)$$

$$S_{30} = 15(100 + (29)(-5))$$

$$\boxed{S_{30} = -675}$$

Male fireflies flash in various patterns to signal location or to ward off predators. Different species of fireflies have different flash characteristics, such as the intensity of the flash, the rate of the flash, and the shape of the flash. Suppose that under certain circumstances, a particular firefly flashes $\boxed{\text{twice}}$ in the first minute, $\boxed{\text{four}}$ times in the second minute, and $\boxed{\text{six}}$ times in the third minute.

a) If this pattern continues, what is the number of flashes in the 30th minute?

$2, 4, 6 \ldots$

$n = 30$

$t_1 = 2$

$d = 2$

$t_n = t_1 + (n-1)d$

$t_{30} = 2 + (30-1)2$

$t_{30} = 2 + (29)2$

$t_{30} = 2 + 58$

$\boxed{t_{30} = 60}$

b) What is the total number of flashes in 30 min?

$S_n = \frac{n}{2}(2t_1 + (n-1)d)$

$S_{30} = \frac{30}{2}(2(2) + (30-1)2)$

$S_{30} = 15(4 + 29(2))$

$S_{30} = 15(4 + 58)$

$S_{30} = 15(62)$

$\boxed{S_{30} = 930}$

PRACTICE TIME!

Pages 27-31 (Q1-6 PICK 3; Q7-15 PICK 3; Q16-23 PICK 1)