NAME:____________________________

U1:L3 Geometric Sequences

$t_n = x$

Below is an example of a geometric sequence:

$y =$ 1, 2, 4, 8, 16, 32, 64, 128, 256, ...

x	y		
1	1	+1	×2
2	2	+2	×2
3	4	+4	×2
4	8		

How can you explain this sequence with $y = mx + b$?

NO it's not linear

In a **Geometric Sequence** each term is found by multiplying the previous term by a constant.

In general we write a Geometric Sequence like this:

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 =$

$$t_1,\ t_1r,\ t_1r^2,\ t_1r^3,\ t_1r^4 \ldots$$

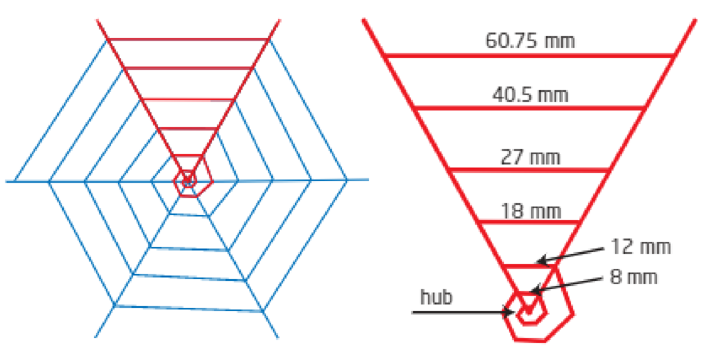
- t_1 is the First term, and
- r is the factor between the terms (called the "common ratio")
- But be careful, r should never = 0 ...When r=0, we get the sequence {a,0,0,...} which is not geometric

This can be simplified to generally write:

$$t_n = t_1 r^{n-1}$$

- n is the number of terms

A geometric sequence can be approximated by the orb web of the common garden spider. A spider's orb web is an impressive architectural feat. The web can capture the beauty of the morning dew, as well as the insects that the spider may feed upon. The following graphic was created to represent an approximation of the geometric sequence formed by the orb web.



Determining t_n

Consider the following sequence…

VERIFY

10, 30, 90, 270, 810, 2430, …

a) Prove that the fourth term is 270. $t_4 = 270$

$r = \times 3$

$$t_n = t_1 r^{n-1}$$
$$t_4 = 10(3^{4-1})$$
$$t_4 = 10(3^3)$$
$$t_4 = 10(27)$$
$$t_4 = 270$$
$$270 = 270 \checkmark$$

b) $\boxed{\text{Find}}$ the tenth term.

$$t_n = t_1 r^{n-1}$$
$$t_{10} = 10(3^{10-1})$$
$$t_{10} = 10(3^9)$$
$$t_{10} = 10(19683)$$
$$t_{10} = 196830$$
$$t_{10} = 196\ 830$$

USA ~~$t_{10} = 196,830$~~

Determining t_1

$t_3 = 54$ $t_6 = -1458$

In a geometric sequence, the third term is 54 and the sixth term is -1458. Determine t_1. ?

★ Find r

WAY #1

$t_n = t_1 r^{n-1}$

$t_6 = t_1 r^{6-1}$

$t_6 = t_1 r^5$

$t_6 = t_3 r^3$

$\frac{-1458}{54} = \frac{54r^3}{54}$

$\sqrt[3]{-27} = \sqrt[3]{r^3}$

$-3 = r$

$t_n = t_1 r^{n-1}$

$t_3 = t_1 r^{3-1}$

$54 = t_1(-3)^2$

$\frac{54}{9} = \frac{t_1(\cancel{9})}{\cancel{9}}$

$6 = t_1$

WAY #2

$t_3 = 54$

$t_6 = -1458$

Solve for t_1

$t_3 = t_1 r^{3-1}$

$54 = t_1 r^2$

$\boxed{\frac{54}{r^2} = t_1}$

Plug t_1 into t_6

$t_6 = t_1 r^{6-1}$

$-1458 = \left(\frac{54}{r^2}\right) r^5$

$\frac{-1458}{54} = \frac{54r^3}{54}$

$\sqrt[3]{-27 = r^3}$

$-3 = r$

Determining r

The modern piano has 88 keys. The frequency of the notes range from A_0, the lowest note, at 27.5 Hz, to C_7, the highest note on the piano, at 4186.009 Hz. The frequencies of these notes approximate a geometric sequence as you move up the keyboard.

★ r

Determine the common ratio of this geometric sequence produced from the lowest key to the fourth (C_1) which his 32.7 Hz.

$t_1 = 27.5$

$n = 4$

$t_4 = 32.7$

$r = ?$

$$t_n = t_1 r^{n-1}$$

$$32.7 = 27.5 r^{4-1}$$

$$\frac{32.7}{27.5} = \frac{\cancel{27.5} r^3}{\cancel{27.5}}$$

$$\sqrt[3]{1.189} = \sqrt[3]{r^3}$$

$$1.059 = r$$

Why "Geometric" Sequence?

Because it is like increasing the dimensions in **geometry**:

- r — a line is 1-dimensional and has a length of r
- r^2 — in 2 dimensions a square has an area of r^2
- r^3 — in 3 dimensions a cube has volume r^3
- ... etc (yes we can have 4 and more dimensions in mathematics).

PRACTICE TIME! PAGES 39 -45 (Q1,3,4,8,9,15, 24,25)