NAME:________________________________

U1:L4 Geometric Series

A geometric series is the terms of a geometric sequence expressed as a sum.

To add geometric sequences we can use…

$$t_1,\ t_1r,\ t_1r^2 \ldots t_1r^{(n-1)}$$

Each term is t_1r^k, where k starts at 0 and goes up to n-1

So…

Add and backwards version

$$S_n = \frac{t_1r^n - t_1}{r-1}$$

Looks the Same as ARITHMETIC ○○ ○ AR or GE

d or r?

Or…

$$\sum_{k=0}^{n-1} t_1r^k$$

Start · Go To · reference for t_n

$\sum_{k=0}^{n-1} t_1r^k$ (called Sigma) means "sum up"…start at bottom value and go to top value, summing up what is after Sigma.

This is also seen as simply S_n…

$$S_n = \frac{t_1(r^n-1)}{r-1};\quad r \neq 1$$

- S_n: the series (sum of "n" terms)
- t_1 is the first term of the series
- r is the common ratio
- n is the number of terms
- $\boxed{r \neq 1}$ means that this doesn't work when the common ratio is 1.

EXAMPLES

Find the sum of the first 4 terms of:

10, 30, 90, 270, ~~810, 2430, …~~

$t_1 \quad t_2 \quad t_3 \quad t_4$

$t_1 = 10$

$r = +3$

$n = 4$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_n = \frac{10(3^4 - 1)}{3 - 1}$$

$$S_n = \frac{10(80)}{2}$$

$$\boxed{S_n = 400}$$

Verify

$400 = 10 + 30 + 90 + 270$

$400 = 400$ ✓

Add up the first $\boxed{\text{10 terms}}$ of the Geometric Sequence that halves each time:

{ 1/2, 1/4, 1/8, 1/16, … }

$n = 10$

$t_1 = \frac{1}{2}$

$r = \frac{1}{2}$

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_n = \frac{\frac{1}{2}\left(\boxed{\left(\frac{1}{2}\right)^{10}} - 1\right)}{\frac{1}{2} - 1}$$

$$S_n = \frac{\frac{1}{2}\left(\frac{1^{10}}{2^{10}} - 1\right)}{-\frac{1}{2}}$$

$$S_n = \frac{\cancel{\frac{1}{2}}\left(\frac{1}{1024} - \frac{1024}{1024}\right)}{-\cancel{\frac{1}{2}}} = -\left(\frac{-1023}{1024}\right) = +1$$

0.999

So, what happens when n is unknown?

Determine the sum of the geometric sequence:

$$\left\{\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \ldots 729\right\}$$

$t_1 = \frac{1}{27}$

$n = ?$

$r = 3$

$t_n = 729$

$t_n = t_1 r^{n-1}$

$729 = \frac{1}{27}(3)^{n-1}$

$\boxed{19683} = (3)^{n-1}$ ★Exponent Laws

$(3)^3 \cdot (3)^6 = (3)^{n-1}$

$3^9 = 3^{n-1}$

$\boxed{10 = n}$

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_n = \frac{\frac{1}{27}(3^{10} - 1)}{10-1} = \frac{\frac{1}{27}(59048)}{9}$

$\boxed{S_n = 242.996}$

An advertising company designs a campaign to introduce a new product to a metropolitan area. The company determines that 1000 people are aware of the product at the beginning of the campaign. The number of new people aware increases by 40% every 10 days during the campaign. How many people will be aware of the campaign after 100 days?

$t_1 = 1000$

$r = 1.4$

$n = 10$

1000, 1400, 1960 ...

×1.4 ×1.4

$S_{10} = \frac{t_1(r^n - 1)}{r-1} = \frac{1000(1.4^{10} - 1)}{1.4 - 1} = \frac{1000(27.93)}{0.4}$

$S_{10} = \frac{27930}{0.4} = \boxed{69825}$

PRACTICE! PAGES 53-57 (Q 1,2,4,9,16,22)