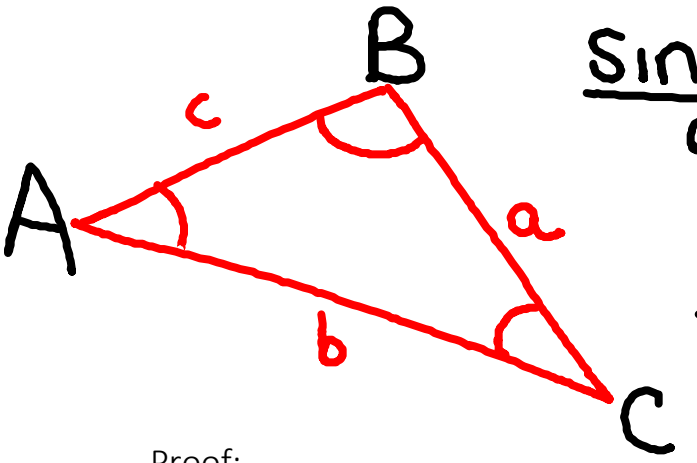
U2:L3 The Sine Law

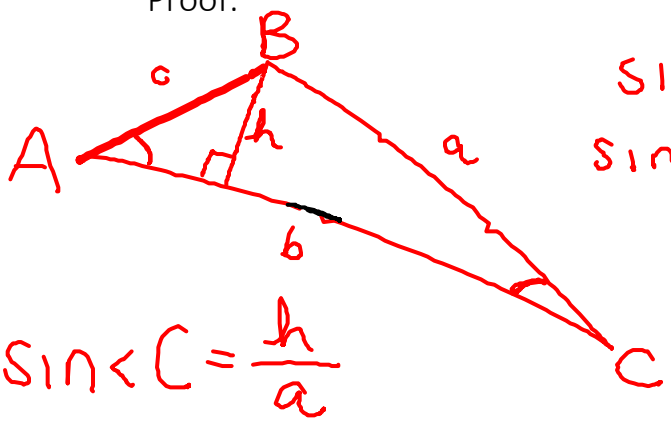
The sine law is the relationship between the sides and angles in and triangle.



$$\frac{\sin\angle A}{a} = \frac{\sin\angle B}{b} = \frac{\sin\angle C}{c}$$

$$\frac{a}{\sin\angle A} = \frac{b}{\sin\angle B} = \frac{c}{\sin\angle C}$$

Proof:



$\sin A = \frac{h}{c}$

$\sin A(c) = h$

$\sin\angle C = \frac{h}{a}$

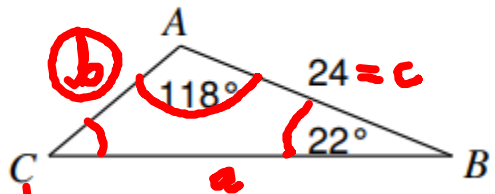
$\sin\angle C(a) = h$

$\frac{\sin A(c)}{a\,c} = \frac{\sin C(a)}{c\,a}$

$\frac{\sin A}{a} = \frac{\sin C}{c}$

Finding side lengths...

1) Find $\overline{AC}$ = b



180 − 118 − 22 = 40°

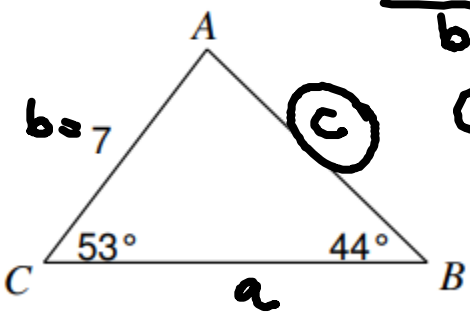
$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin B\,(c)}{\sin C} = b$

$\frac{\sin(22°)(24)}{\sin(40°)} = b$

$13.99 = b$

2) Find $\overline{AB}$



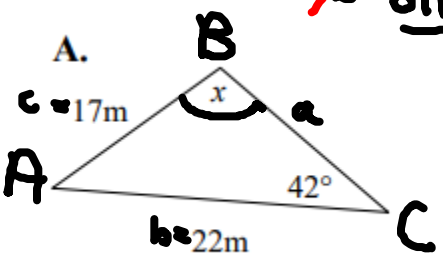
$\frac{\sin B}{b} = \frac{\sin C}{c}$ OR $\frac{b}{\sin B} = \frac{c}{\sin C}$

$c = \frac{(b)\sin C}{\sin B} = \frac{7\sin 53°}{\sin 44°}$

$c = 8.05$

Finding angles...

A.



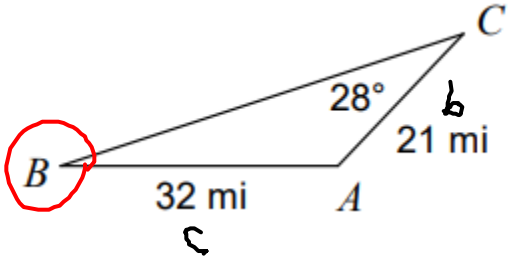
$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\sin B = \frac{\sin C\,(b)}{c} = \frac{\sin 42°(22)}{17}$

$\frac{\sin B}{\sin} = \frac{0.8659}{\sin}$

$B = 59.99°$

Find $< ABC$



$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\sin B = \frac{\sin C(b)}{c} = \frac{\sin 28°(21)}{32}$

$B = 17.94°$

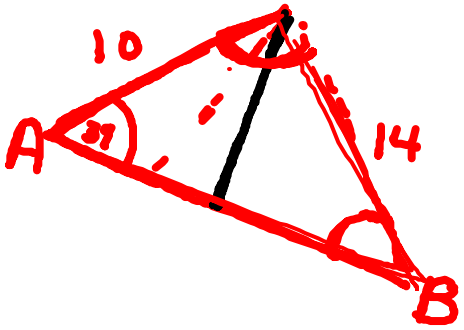
The Ambiguous Case

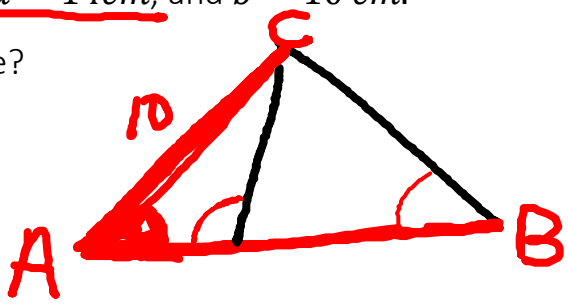
Because two sides and an angle opposite one of the sides is known, there are either…

- two possible situations
- one solution
- no solutions

Suppose ΔABC where $< A = 39°$, $a = 14cm$, and $b = 10\ cm$.

What would these triangles look like?





$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\sin B = \frac{\sin A(b)}{a}$$

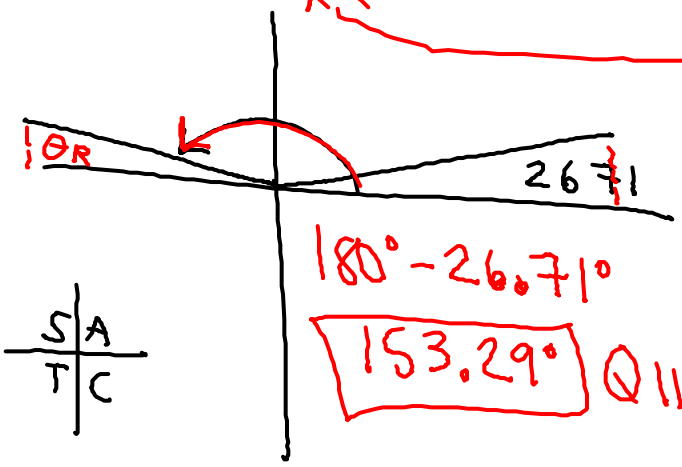
$$\sin B = \frac{\sin(39°)(11)}{14}$$

Q1 $\boxed{< B = 26.71°}$

Then solve for your possibilities…

$< C = 180° - 39° - 26.71°$

$< C = 114.29°$

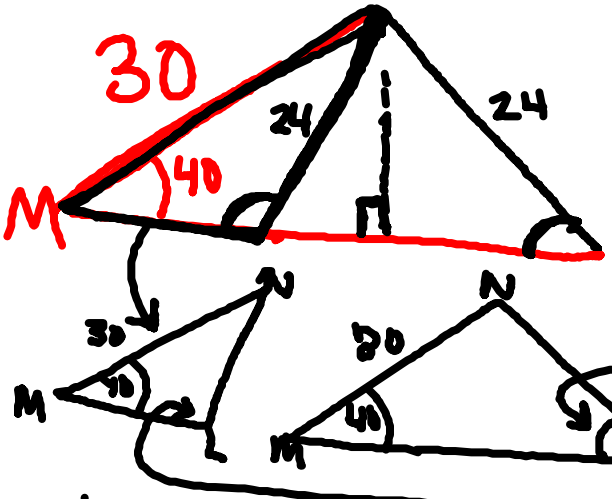


$180° - 153.29° - 39°$

$< C = -12.92°$

Suppose ΔLMN where $< M = 40°$, $l = 30cm$, and $m = 24\ cm$.

Solve for all possibilities of $< L$...



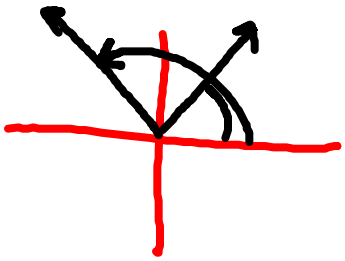
$$\frac{\sin L}{l} = \frac{\sin M}{m}$$

$$\sin L = \frac{\sin M\,(l)}{m}$$

$$\sin L = \frac{\sin(40°)30}{24}$$

Q1 $L = 53\,46°$

Q11 → $180° - 53\,46° = 126\,54°$



Check ΔLMN

$40° + 126.54° = 166\,54°$

Yes! Two possibilities

PRACTICE: Pages 108-113 Qs 1, 2,3, 6.