

NAME: answers

U4:L4 RATIONAL EXPRESSIONS

Fill in the following lesson with help from pages 310 - 317 in your textbook or my website www.burnspvw.weebly.com

A rational expression is an algebraic fraction with a numerator and a denominator that are polynomials.

Examples:

$\frac{1}{x}$	$\frac{m}{m+3}$	$\frac{y^2 + 2y}{3y - 1}$
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monomial (1 TERM) points to $\frac{1}{x}$. *binomial (2 TERMS)* points to $\frac{m}{m+3}$.

*** If a polynomial is **not a fraction** it is still a rational expression, because it can be

written with a denominator of **1**. Example:

$3x - 5$	$=$	$\frac{3x - 5}{1}$
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An arrow points from the **1** in the text above to the denominator of the fraction on the right.

*** Remember: denominators can be any integer or polynomial **except for zero**.

What happens if you plug the following into your calculator?

$\frac{32}{0} = \text{ERR}$	$\frac{400}{0} = \text{ERR}$	$9 \div 0 = \text{ERR}$
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NON-PERMISSIBLE VALUES

Whenever working with rational expressions, you need to find the **non-permissible values**. These are values for the variable that make the denominator **zero**.

Example:

$$\frac{x + 2}{x - 3}$$

$$x - 3 \neq 0$$

$$x \neq 3$$

denominator as zero is "undefined". (An arrow points from this text to the denominator $x - 3$ in the example above.)

Determine the non-permissible values:

a) $\frac{5t}{4sr^2}$

$4sr^2 \neq 0$
 $4s \neq 0$ or $r^2 \neq 0$
 $s \neq 0$ or $r \neq 0$

b) $\frac{3x}{x(2x-3)}$

$x(2x-3) \neq 0$
 $x \neq 0$ or $2x-3 \neq 0$
 $2x \neq 3$
 $x \neq 3/2$

$x \neq 0$
 or
 $x \neq 3/2$

c) $\frac{2p-1}{p^2-p-12}$

Factor Trinomial
 $p^2 - p - 12 \neq 0$
 $(p-4)(p+3) \neq 0$
 $p \neq 4$ or $p \neq -3$

$\begin{matrix} \otimes = -12 \\ \oplus = -1 \end{matrix} \left. \vphantom{\begin{matrix} \otimes \\ \oplus \end{matrix}} \right\} -4, +3$

Equivalent Rational Expressions

You can **multiply** or **divide** a rational expression by 1 and not change its value. This will create an **equivalent expression**.

ANY RATIO of One!
 1 or $\frac{1}{1}$
 or $\frac{5}{5}$ or $\frac{x}{x}$
 ...

Example:

$\frac{7s}{s-2} \times \left(\frac{s}{s}\right) = \frac{7s \cdot s}{(s-2)s} = \frac{7s^2}{s(s-2)}$

OR $\frac{7s}{s-2} \times \frac{(s+2)}{(s+2)} = \frac{7s(s+2)}{(s-2)(s+2)}$

Both Equivalent

What are the non-permissible values of "s"?

$s-2 \neq 0$
 $s \neq 2$

$s \neq 0$
 $s \neq 2$

$s \neq 2$ or $s \neq -2$
 $s \neq \pm 2$

OR
 $\frac{7s}{s-2}$

OR
 $\frac{7s^2}{s(s-2)}$

OR
 $\frac{7s(s+2)}{(s-2)(s+2)}$

Simplifying Rational Expressions

To simplify a rational expression, divide both the numerator and denominator by any common factors.

Examples:

$$a) \frac{9}{12} = \frac{\cancel{3} \cdot 3}{\cancel{3} \cdot 4} = \boxed{\frac{3}{4}}$$

$$b) \frac{m^3 t}{m^2 t^4} = \frac{\cancel{m} \cdot \cancel{m} \cdot m \cdot \cancel{t}}{\cancel{m} \cdot \cancel{m} \cdot \cancel{t} \cdot t \cdot t \cdot t} = \frac{m}{t \cdot t \cdot t} = \boxed{\frac{m}{t^3}} \quad \begin{matrix} m \neq 0 \\ t \neq 0 \end{matrix}$$

"m" is included in NON-PERMISSIBLE because we started with an 'm' denominator

When a rational expression is in simplest form, the numerator and denominator should have no common factors (other than 1).

Try it:

$$a) \frac{(3x-6)}{2x^2+x-10} = \frac{3(x-2)}{2x^2-4x+5x-10} = \frac{3(x-2)}{2x(x-2)+5(x-2)}$$

$$= \frac{3 \cdot \cancel{(x-2)}}{(2x+5) \cdot \cancel{(x-2)}} = \boxed{\frac{3}{2x+5}}$$

RESTRICTIONS \rightarrow $2x+5 \neq 0$
 $2x \neq -5$
 $\boxed{x \neq -5/2}$
 and $x-2 \neq 0$
 $\boxed{x \neq 2}$

$$b) \frac{(1-t)}{t^2-1}$$

$$\frac{1-t}{(t-1)(t+1)} = \frac{-1 \cdot \cancel{(t-1)}}{\cancel{(t-1)}(t+1)} = \boxed{\frac{-1}{t+1}}$$

RESTRICTIONS \rightarrow
 $t-1 \neq 0$ OR $t+1 \neq 0$
 $t \neq 1$ OR $t \neq -1$
 $\therefore \boxed{t \neq \pm 1}$

Multiple Non-Permissible Values

Consider:

$$\frac{16x^2 - 9y^2}{8x - 6y}$$

a) How would you find the non-permissible values for x ?

$$\begin{aligned}8x - 6y &\neq 0 \\8x &\neq 6y \\x &\neq \frac{6y}{8}\end{aligned}$$

$$x \neq \frac{3y}{4}$$

So... if $y = 1$
 $x \neq \frac{3(1)}{4} = \frac{3}{4} \quad \left(\frac{3}{4}, 1\right)$

if $y = 2$
 $x \neq \frac{3(2)}{4} = \frac{6}{4} = \frac{3}{2} \quad \left(\frac{3}{2}, 2\right)$

if $y = 3$
 $x \neq \frac{3(3)}{4} = \frac{9}{4} \quad \left(\frac{9}{4}, 3\right)$

etc...

b) Simplify the expression

$$\frac{6y}{8} \Rightarrow \frac{3y}{4}$$

c) Evaluate the expression for $x = 2.6$ and $y = 1.2$

Plug into original equation.

$$\frac{16x^2 - 9y^2}{8x - 6y} = \frac{16(2.6)^2 - 9(1.2)^2}{8(2.6) - 6(1.2)}$$

$$= \frac{95.2}{13.6}$$

$$= 7$$

PRACTICE: Page 317 – Questions 1(a,b,c,d), 4 (a,b,c), 6 (a,b), 7, 8 (a,b,c) and 13

U4:L5 MULTIPLY and DIVIDE RATIONAL EXPRESSIONS

Multiplying rational expressions is like multiplying any rational fraction:

- 1) Multiply the numerators
- 2) Multiply the denominators
- 3) Simplify the fraction

Examples:

$$a) \frac{5}{8} \times \frac{4}{15} = \frac{5 \times 4}{8 \times 15} = \frac{20}{120} = \frac{1}{6}$$

$$\text{OR } \frac{\cancel{5} \times \cancel{4}}{2 \times \cancel{4} \times \cancel{3} \times 3} = \frac{1}{2 \times 3} = \left(\frac{1}{6} \right)$$

$$b) \frac{4x^2}{3xy} \times \frac{y^2}{8x}$$

$$\frac{4x^2(y^2)}{3xy(8x)} = \frac{4x^2y^2}{24x^2y} = \frac{\cancel{1}x \cdot \cancel{x} \cdot \cancel{y} \cdot y}{\cancel{6}x \cdot \cancel{x} \cdot y} = \left(\frac{y}{6} \right)$$

Try it:

$$\frac{a^2 - a - 12}{a^2 - 9} \times \frac{a^2 - 4a + 3}{a^2 - 4a}$$

SIMPLIFY FIRST!

$$\frac{(a^2 - a - 12) \times (a^2 - 4a + 3)}{(a^2 - 9) \times (a^2 - 4a)} \quad \text{OR} \quad \frac{\cancel{(a-4)} \cancel{(a+3)}}{\cancel{(a-3)} \cancel{(a+3)}} \times \frac{\cancel{(a-3)}(a-1)}{a \cancel{(a-4)}}$$

$$\left(\frac{a-1}{a} \right)$$

Dividing rational expressions follows similar steps as dividing fractions too!

Method One: Use a common denominator

a) $\frac{5}{3} \div \frac{1}{6}$

$$\frac{10}{6} \div \frac{1}{6} = \frac{10}{1} = 10$$

b) $\frac{3x^2}{y^2} \div \frac{x}{y}$

$$\frac{3x^2}{y^2} \div \frac{xy}{y^2} = \frac{3x^{\cancel{2}}}{\cancel{xy}} = \left(\frac{3x}{y}\right) \quad \begin{array}{l} x \neq 0 \\ y \neq 0 \end{array}$$

Method Two: MULTIPLY the RECIPROCAL (flipped fraction)

a) $\frac{5}{3} \div \frac{1}{6}$

$$\frac{5}{3} \times \frac{6}{1} = \frac{30}{3} = 10$$

OR

$$\frac{5 \times \cancel{2} \times \cancel{3}}{\cancel{3} \times 1} = \frac{10}{1} = 10$$

b) $\frac{3x^2}{y^2} \div \frac{x}{y}$

$$\frac{3x^2}{y^2} \times \frac{y}{x} = \frac{3x^2y}{y^2x} = \frac{3x \cdot \cancel{x} \cdot \cancel{y}}{\cancel{y} \cdot \cancel{y} \cdot \cancel{x}} = \left(\frac{3x}{y}\right)$$

Try it with whichever method you prefer:

$$1) \frac{x^2-4}{x^2-4x} \div \frac{x^2+x-6}{x^2+x-20}$$

$$\frac{(x-2)(x+2)}{x(x-4)} \div \frac{x^2+3x-2x-6}{x^2-4x+5x-20}$$

$$\frac{(x-2)(x+2)}{x(x-4)} \div \frac{(x+3)(x-2)}{(x-4)(x+5)}$$

$$\frac{\cancel{(x-2)}(x+2)}{x \cancel{(x-4)}} \times \frac{\cancel{(x-4)}(x+5)}{(x+3)\cancel{(x-2)}}$$

$$\frac{(x+2)(x+5)}{x(x+3)}$$

$$\begin{aligned} x &\neq -5 & x &\neq 2 \\ x &\neq +4 & x &\neq -3 \\ x &\neq 0 \end{aligned}$$

ALL the RESTRICTIONS from ALL DENOMINATORS

What are the non-permissible values of x?

$$x \neq -5, 4, 0, 2, -3.$$

$$2) \frac{2m^2-7m-15}{2m^2-10m} \div \frac{4m^2-9}{6} \times (3-2m)$$

$$\frac{2m^2+3m-10m-15}{2m(m-5)} \div \frac{(2m+3)(2m-3)}{6} \times (3-2m)$$

$$\frac{\cancel{2m+3} \cancel{(m-5)}}{\cancel{2m} \cancel{(m-5)}} \times \frac{3}{6} \times \frac{-1 \cancel{(2m-3)}}{1}$$

$$\frac{3(-1)}{m(1)} = \frac{-3}{m}$$

What are the non-permissible values of m?

$$2m \neq 0$$

$$m-5 \neq 0$$

$$2m+3 \neq 0$$

$$2m-3 \neq 0$$

$$m \neq 0$$

$$m \neq 5$$

$$2m \neq -3$$

$$2m \neq 3$$

$$m \neq -3/2$$

$$m \neq 3/2$$

$$m \neq 0, 5, -3/2, 3/2$$

PRACTICE: Page 327 – Questions 1, 2, 4, 8, 15, 16.

Need more step-by-step examples??
email / message me on Teams ☺

