11 PRECAL

Unit 6: Booklet 2

Equations and Inequalities

June 2nd - June 9th

NAME: Answer Key

U6:L3 Linear Inequalities (2 Variables)

Fill the following lesson with help from pages 465-471 or the filled notes at www.burnspvw.weebly.com

Inequality Review

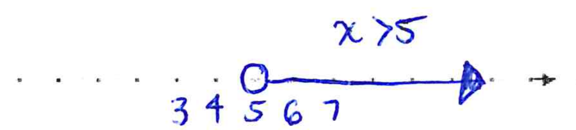
Remember that inequalities are mathematical phrases that compare values.

Inequalities use the following symbols:

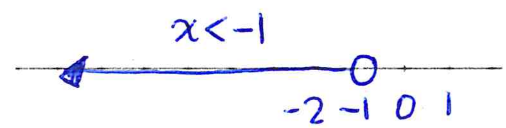
$x >$	LARGER than / MORE than
$x <$	SMALLER than / LESS than
$x \geq$	Larger than OR equal to
$x \leq$	Smaller than OR equal to

Remember also, that these can be represented graphically on a number line:

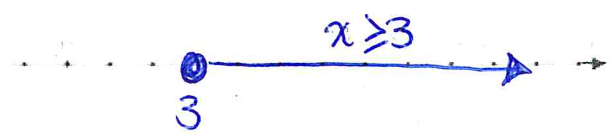
$>$ Greater than



$<$ Less than



$\geq$ Greater than or equal to



$\leq$ Less than or equal to

Inequalities that are not also "equal to" are non-inclusive and are represented by a circle that is not filled. O→ OR ←O

Inequalities that are also "equal to" are inclusive and are represented by a filled circle.

●→ OR ←●

SLOPE DUDE

Linear Inequalities

m= SLOPE

b= y-intercept

Linear Inequalities are based on the linear relation y=mx+b (remember that all linear relations follow this format).

A linear inequality with two variables must include:

- X variable
- Y variable
- Inequality symbol

Below, A, B, are real number coefficients to the x and y variables.

C is a real number.

The possibilities for linear inequalities, are therefore:

$Ax + By > C$
$Ax + By < C$
$Ax + By \geq C$
$Ax + By \leq C$

$Ax + By = C$ and the above inequalities are a re-arranged version of $y = mx + b$

Any ordered pair (x,y) is a solution to the inequality if, when plugged into the above inequalities, the inequality is true.

There will be multiple solutions.

The set of points that satisfy the linear inequality is called a solution set or solution region.

Example:

Which of the following ordered pairs satisfy the solution set for $\boxed{3x + 2y > 10}$?

a) $(1,4)$ $3(1) + 2(4) > 10$

$3 + 8 > 10$

$11 > 10$

True so... YES!

b) $(-5, -12)$

$3(-5) + 2(-12) > 10$

$-15 + (-24) > 10$

$-39 > 10$

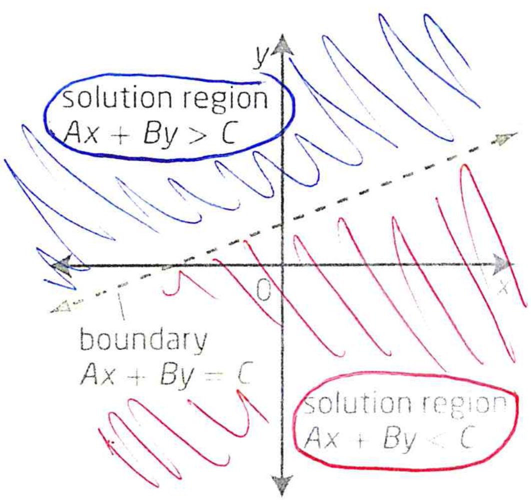
not true, so... NO!

Graphing Linear Inequalities

One way of solving linear inequalities is to graph the linear relation on a Cartesian Plane.

The line graphed with be the linear equation $Ax + By = C$. This line is called the boundary. This line divides the Cartesian plane into two regions:

- One region $Ax + By < C$ is true.
- One region $Ax + By > C$ is true.



Boundary may or may not be part of the solution set.

Boundary is drawn as a SOLID line if it is part of the solution set ($\leq$ *or* $\geq$)

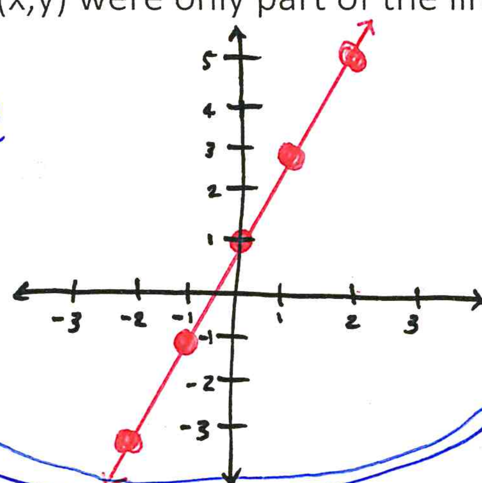
Boundary is drawn as a DASHED line if it is not part of the solution set ($<$ *or* $>$)

Remember (from grade 9) that when you first studied linear relations, the ordered pairs (x,y) were only part of the linear relation if they were on the line.

FOR Example

$y = 2x + 1$

	x	y
$(-2, -3)$	-2	-3
$(-1, -1)$	-1	-1
$(0, 1)$	0	1
$(1, 3)$	1	3
$(2, 5)$	2	5



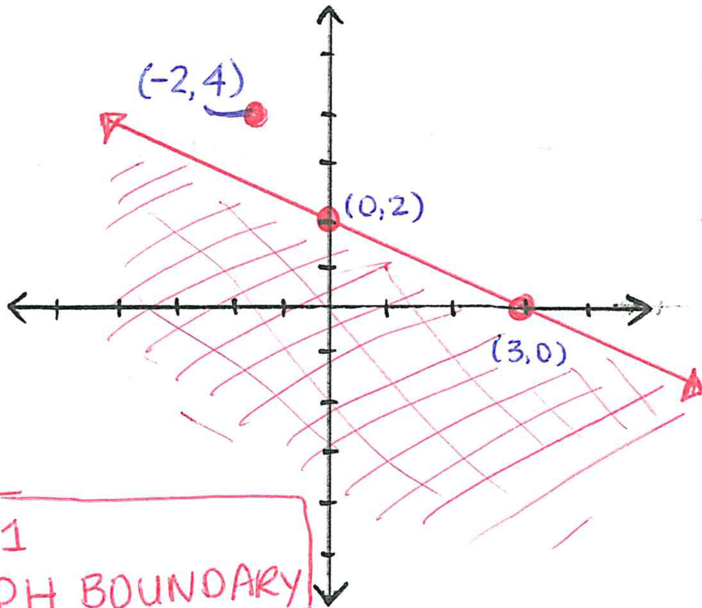
These are the only points that satisfy

$y = 2x + 1$

Now, our solutions with linear inequalities are the regions above or below the line. This can include the line or not include the line.

Example 1:

Is the point $(-2,4)$ part of the solution set of $2x + 3y \leq 6$?



* Boundary is SOLID line because $\leq$ is inclusive.

* Shaded region is:
$2x + 3y \leq 6$

* Not Shaded region is:
$2x + 3y \geq 6$

STEP 1
GRAPH BOUNDARY

one option to graph is by finding the intercepts:

when $x = 0$

$2x + 3y = 6$
$2(0) + 3y = 6 \quad (0,2)$
$0 + 3y = 6$
$y = 2$

when $y = 0$

$2x + 3y = 6$
$2x + 3(0) = 6 \quad (3,0)$
$2x = 6$
$x = 3$

STEP 2
DETERMINE REGIONS

use a TEST POINT to see what region it fits in.
so... (0,0) for example

$2x + 3y \leq 6$
$2(0) + 3(0) \leq 6$
$0 \leq 6$ ✓

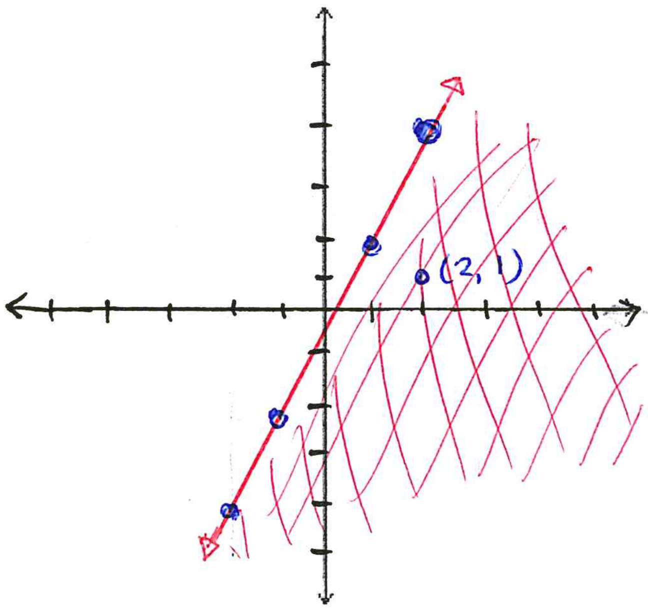
Because this is TRUE, and (0,0) is UNDER the line, we shade under the boundary as the solution ☺

STEP 3
Point included?

Plug (-2,4) into the equation OR find it on the graph to see what region it is in. NOT INCLUDED!

Example 2:

Graph $10x - 5y > 0$



* Boundary $y = 2x$
* under boundary: $10x - 5y > 0$
* over boundary $10x - 5y < 0$

1: GRAPH BOUNDARY

Another method is to rearrange to $y = mx + b$

$10x - 5y > 0$

$10x > 5y$

$2x > y$ → $y = 2x$ is Boundary

↳ $m = \text{slope} = 2$

2: REGIONS W/ TEST POINT

use any Test point!

(2, 1) (UNDER)

$10x - 5y > 0$

$10(2) - 5(1) > 0$

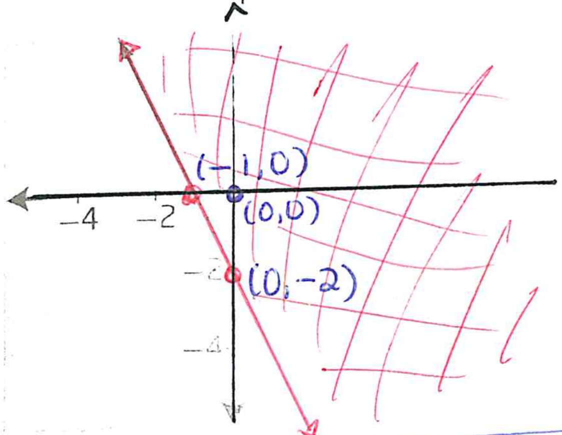
$20 - 5 > 0$

$15 > 0$ ✓

so under is True!

Example 3:

Write the inequalities that the following graphs represent:



① Determine boundary line w/ $y=mx+b$

$b=-2$

$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{0-(-2)}{-1-0} = \frac{2}{-1}$

$m=-2$

$\boxed{y=-2x-2}$

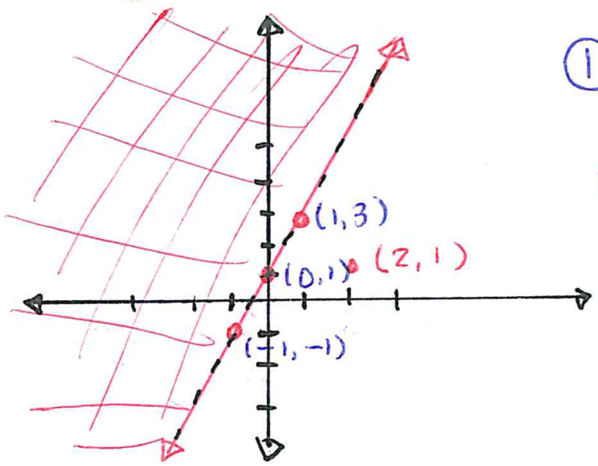
② Determine Inequality Symbol

SOLID Line, so inclusive $\cancel{>}$ $\cancel{<}$

use a test point:
(0,0)

$y=-2x-2$
$0=-2(0)-2$
$0=-2$
$0 > -2$

$\boxed{y > -2x-2}$



① Determine boundary Line

y-intercept = 1 so $b=1$

Find SLOPE (m) with any two points

slope $= m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$

(1,3) and (0,1) $m = \frac{3-1}{1-0} = \frac{2}{1} = 2$

$\boxed{y=2x+1}$

② Determine Inequality Symbol

Dashed Line = not inclusive $\cancel{\geq}$ $\cancel{\leq}$

use a Test point to find $>$ or $<$

(2,1)
$y=2x+1$
$1=2(2)+1$
$= 4+1$
< 5

$\boxed{y < 2x+1}$

PRACTICE: Page 472, Questions 1a, 1b, 3a, 3b, 3c, 4a, 4b, 9

U6:L4 Quadratic Inequalities (1 Variable)

Fill the following lesson with help from pages 476-484 or the filled notes at

Just as you used the y=mx+b formula to work with linear inequalities, quadratic inequalities work from the standard quadratic formula:

$$ax^2 + bx + c = 0$$

Quadratic inequalities can be:

$ax^2 + by + c > 0$
$ax^2 + by + c < 0$
$ax^2 + by + c \geq 0$
$ax^2 + by + c \leq 0$

The variables a, b, and c are real numbers, and $a \neq 0$

The solution set to a quadratic inequality can have:

- NO values
- ONE value
- INFINITE values

There are multiple ways to solve quadratic inequalities:

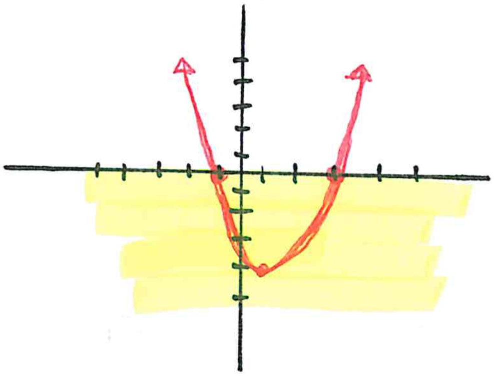
- GRAPHICALLY (with help from Desmos!)
- ROOTS + TEST POINTS.
-

Example 1:

⊝ sign graph is the BOUNDARY

Solve $x^2-2x-3\le 0$ → $x^2-2x-3=0$

a) Graphically: (Use Desmos)



☆ for $x^2-2x-3 \le 0$ LOOK for all values of x that the graph is BELOW 0.

→ when x is between -1 and $+3$

$\therefore$ $\boxed{\{x \mid -1 \le x \le 3,\ x \in \mathbb{R}\}}$ ☆

b) Roots and Test Points:

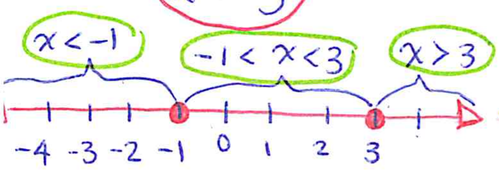
Solve $x^2-2x-3=0$ to find <u>ROOTS</u>

$x^2-2x-3=0$

$(x+1)(x-3)=0$

$x=-1$

$x=3$



INTERVAL	$x<-1$	$-1<x<3$	$x>3$
TEST POINT	-2	0	5
	$x^2-2x-3\le 0$ $(-2)^2-2(-2)-3\le 0$ $4-(-4)-3\le 0$ $4+4-3\le 0$ $8-3\le 0$ $5\le 0$	$(0)^2-2(0)-3\le 0$ $0-0-3\le 0$ $-3\le 0$	$(5)^2-2(5)-3\le 0$ $25-10-3\le 0$ $15-3\le 0$ $12\le 0$
TRUE?	NO.	YES!	NO.

☆ $\boxed{\{x \mid -1 \le x \le 3,\ x \in \mathbb{R}\}}$

Test Points are whatever # you want to use that is IN that interval

Both methods lead to the same answer :)

Example 2:

Solve $-x^2 + x + 12 < 0$

Roots and Test Points:

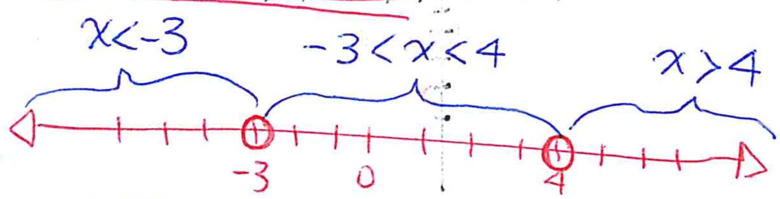
① **Solve** $-x^2 + x + 12 = 0$ to find ROOTS.

$$-x^2 + x + 12 = 0$$
$$-1(x^2 - x - 12) = 0$$
$$x^2 - x - 12 = 0$$
$$(x+3)(x-4) = 0$$

$x = -3$

$x = +4$

② **Intervals**



*NOTE THAT CIRCLE ARE NOT FILLED, BECAUSE < IS NON-inclusive!

③ **Test Points**

INTERVAL	$x < -3$	$-3 < x < 4$	$x > 4$
TEST	-4	0	5
	$-x^2 + x + 12 < 0$ $-(-4)^2 + (-4) + 12 < 0$ $-16 - 4 + 12 < 0$ $-20 + 12 < 0$ $-8 < 0$	$-x^2 + x + 12 < 0$ $-0^2 + 0 + 12 < 0$ $12 < 0$	$-5^2 + 5 + 12 < 0$ $-25 + 5 + 12 < 0$ $-20 + 12 < 0$ $-8 < 0$
TRUE?	YES!	NO.	YES!

*The test values are random, whatever values satisfy the interval.

$$\therefore \{x \mid x < -3 \text{ or } x > 4,\ x \in \mathbb{R}\}$$

Example 3:

Solve $2x^2 - 7x > 12$ with the "Roots and Test Points" method:

① Find Roots by re-arranging to $ax^2+bx+c=0$

$2x^2-7x-12=0$

* use Quadratic Formula to solve.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

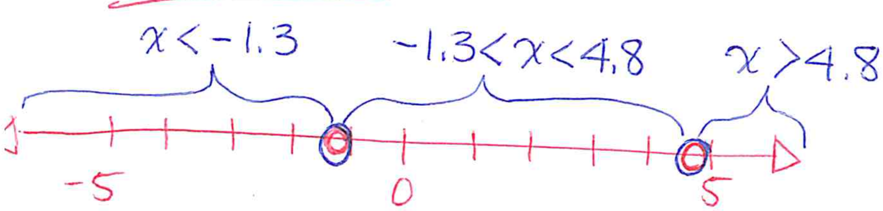
$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(2)(-12)}}{2(2)}$$

$$x=\frac{+7\pm\sqrt{49-(-96)}}{4}$$

$$x=\frac{+7+\sqrt{145}}{4} \quad \text{or} \quad x=\frac{+7-\sqrt{145}}{4}$$

$x\approx 4.8$ $\qquad$ $x\approx -1.3$

② Intervals.



Circles are NOT filled because original inequality is $>$ not-inclusive.

③ Test Points

INTERVAL	TEST		TRUE?
$x<-1.3$	-2	$2x^2-7x>12$ $2(-2)^2-7(-2)>12$ $2(4)-(-14)>12$ $8+14>12$ $22>12$	YES!
$-1.3<x<4.8$	0	$2x^2-7x>12$ $2(0^2)-7(0)>12$ $0-0>12$ $0>12$	NO
$x>4.8$	5	$2x^2-7x>12$ $2(5)^2-7(5)>12$ $50-35>12$ $15>12$	YES!

Final Answer ⇒ $\{x \mid x<-1.3 \text{ or } x>4.8,\ x\in\mathbb{R}\}$

PRACTICE: Page 484, Questions 1, 2, 3a, 3b, 4b, 7a, 7b